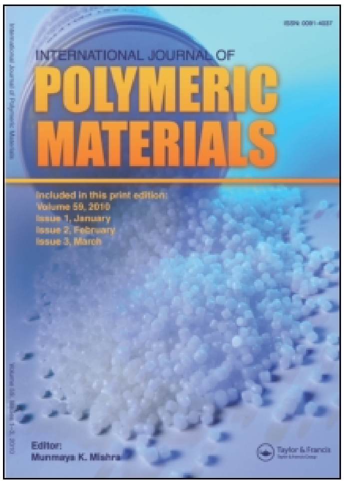

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Theoretical Analysis on Flow of Polymer Melts in Screw Die

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Flow of polymer melt in screw dies is theoretically analyzed by the broken section method with the uniformity of the extrudates. An isothermal, laminar and steady state power law fluid is assumed. The analysis is discussed in two parts, i.e., screw flow and die slit flow. A way of computer calculation by means of a method of iteration is presented by considering volume balance between screw flow and slit flow.

An ideal screw die is one in which pressure distribution is constant along the screw axis, i.e., the shape of the die slit is constant along the axis and the screw is such that the depth of screw channel decreases almost linearly.

INTRODUCTION

Theoretical analyses on die design of plastic sheets and films have been attempted by Carley,[1] Weeks,[2] Pearson,[3,4] McKelvey and Ito[5] and other researchers. This paper presents an analysis on the flow of a power law fluid with a flow index n (≤ 1) in a screw die,[6,7,8,9] Figure 1, by broken section method, and describes a computational procedure for uniformalizing the thickness of the extrudate.

The screw die consists of a die slit and a die screw inserted in a die manifold connected to an extruder, in alignment with the axis of the extruder screw. As shown in Figure 2, molten polymer from the extruder flows through the entrance of the screw die, $\lambda = 0$, into the die, where it is divided into two parts; one is forcibly dragged and conveyed toward the front end of the screw axis, or the λ axis, by the rotation of the die screw. Simultaneously the other is extruded by the fluid pressure toward the die lips located at right angles to the λ axis.

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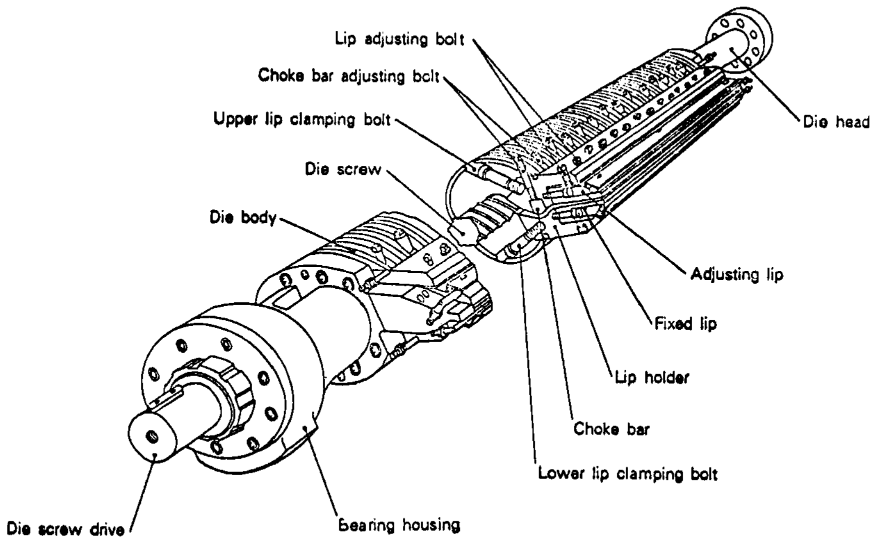

FIGURE 1 Construction of screw die.

In order to analyze the uniformity of the sheet/film, a die screw with a die slit having a geometry varying along the λ axis is broken into M equal sections in parallel with the flow direction of the extrudate.

For simplifying the analysis, the following assumptions are made: molten polymer is an incompressible power law fluid; the flow is steady, laminar and isothermal; there occurs no slipping at the wall; further the interaction between both screw and slit flows can be neglected as well as the various entry effects.

FLOW IN DIE SCREW

As shown in Figure 2, polymer melt at a volumetric flow rate Q_0 enters the die at $x = 0$ under hydrostatic pressure P_0 and flows in the λ direction. Simultaneously, there is a slit flow through the die slit perpendicular to the λ axis.

The analysis is made on the assumption that the screw channels which are sufficiently smaller than the screw radius are approximately represented by parallel-plate models. The coordinates are; the cross channel direction is on the x axis, the channel height direction on the y axis, and the down channel direction on the z axis.

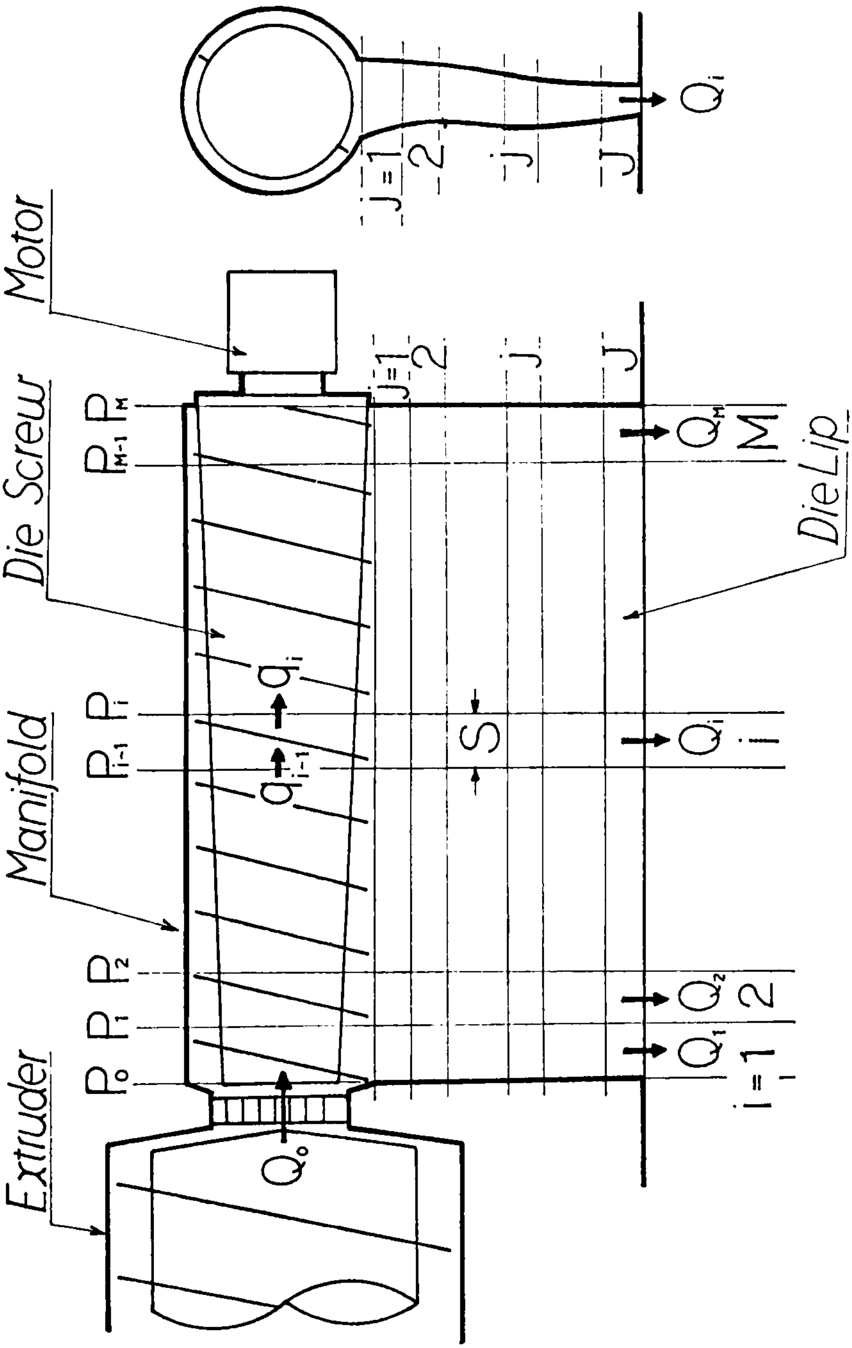

FIGURE 2 Flow in screw die.

The complicated flow of power law fluid in screw channels is analyzed in the following four approaches:

1 Modified non-Newtonian flow in rectangular channel

Refer to Figure 3, if the Newtonian viscosity is replaced with an average non-Newtonian viscosity in order to extend the well-known Newtonian fluid formulas[10,11] to the expressions on non-Newtonian screw characteristics, the following equation of volumetric flow rate is obtained[12]:

$$\hat{Q} = \alpha N \pm \beta\dot{\gamma}^{\circ}\left(\frac{n}{n+1}\right)^{1-n}\left\{\frac{\sin\theta}{\eta^{\circ}\dot{\gamma}^{\circ}}\left(\frac{H}{2}\right)^{1-n}\right\}^{1/n}\left|\frac{\Delta P}{\Delta\lambda}\right|^{1/n} \quad (1)$$

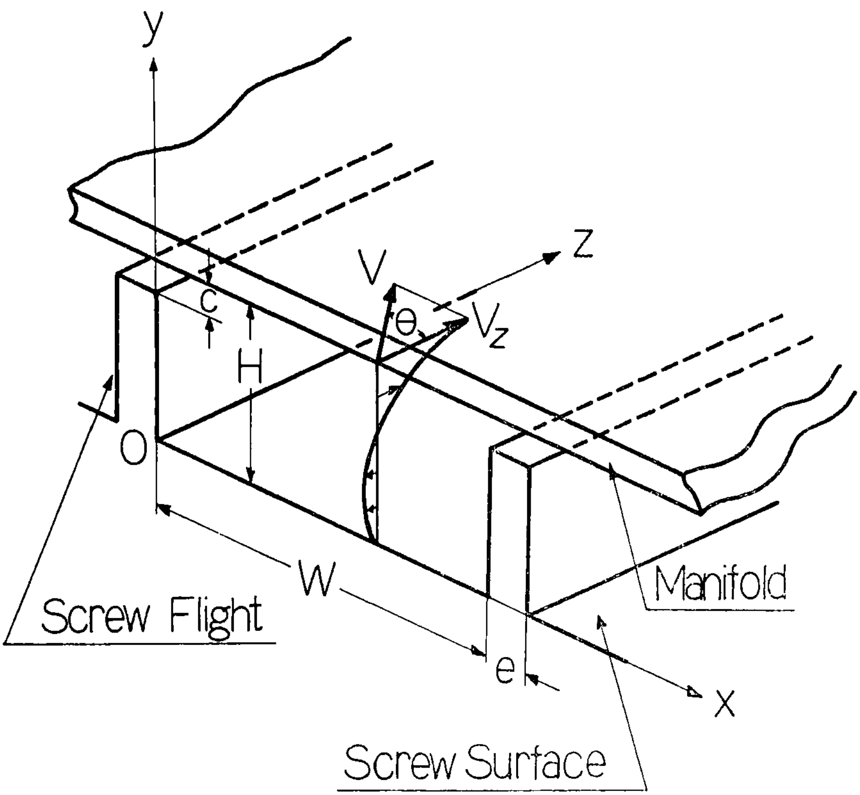

FIGURE 3 An idealized rectangular cross-section of a screw channel.

where

$$\alpha = \frac{\pi^2 D^2 H \sin\theta\cdot\cos\theta}{2}\left(1 - \frac{2C}{D} - \frac{em}{\pi D \sin\theta}\right)\left(1 - \frac{C}{H}\right)^2\cdot F_D.$$

$$F_D = \frac{16W}{\pi^3 H}\sum_{g=1,3,5,\ldots}^{\infty}\left(\frac{1}{g}\right)^3\cdot\tanh\left(\frac{g\pi H}{2W}\right)$$

$$\beta = \frac{\pi D H^3 \sin\theta}{12}\left(1 - \frac{2C}{D} - \frac{em}{\pi D \sin\theta}\right)\left\{1 + \left(\frac{C}{H}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta.\cos\theta}\right)^2\right\}F_P$$

$$F_P = 1 - \frac{192H}{\pi^5 W}\sum_{g=1,3,5,\ldots}^{\infty}\left(\frac{1}{g}\right)^5\cdot\tanh\left(\frac{g\pi W}{2H}\right)$$

$$W = \frac{\pi D \sin\theta}{m}\left(1 - \frac{2C}{D} - \frac{em}{\pi D \sin\theta}\right) \tag{2}$$

and N is the frequency of screw rotation, $(\Delta P/\Delta\lambda)$ is the pressure gradient in the screw axial direction, η° is the apparent non-Newtonian viscosity at shear rate $\dot{\gamma}^\circ$ in the standard state, D is the diameter of the manifold, H is the depth of the channel, W is the width of the channel, e is the flight width, C is the flight clearance, θ is the helix angle of the screw and m is the number of channels arranged in parallel. And the sign of the second term at the right hand side is positive when $(\Delta P/\Delta\lambda) < 0$ or negative when $(\Delta P/\Delta\lambda) > 0$.

The total power $\overline{\overline{HP}}$ required for driving the screw is equal to the sum of the power required to convey polymer melt and the power that is consumed in the screw clearance. The power requirement for a Newtonian fluid[13] is extended to the non-Newtonian fluid. When the width of the channel is constant along the down-channel direction, the power equation for the non-Newtonian fluid can be expressed[14,15]:

$$\frac{\overline{\overline{HP}}}{t} = \xi\eta^\circ\left(\frac{\pi D\cdot\cos\theta}{\dot{\gamma}^\circ H}\right)^{n-1}\cdot N^{n+1}\cdot\Delta\lambda + \zeta N\cdot\Delta P \tag{3}$$

where

$$\xi = \frac{\pi^3 D^3}{H}\left\{1 + 3\sin^2\theta + \frac{e}{W}\left(\frac{H}{C}\right)^n\right\}\left(1 - \frac{2C}{D} - \frac{em}{\pi D \sin\theta}\right)$$

$$\zeta = \frac{\pi^2 D^2 H \sin\theta\cdot\cos\theta}{2}\left(1 - \frac{2C}{D} - \frac{em}{\pi D\cdot\sin\theta}\right)$$

and t is a coefficient for conversion of torque into horse-power.

2 Modified non-Newtonian flow in a rectangular channel having rounded corners on both sides

As shown in Figure 4, the flow of a modified non-Newtonian fluid in a channel is analyzed in two flow regions, a channel with a rounded corner ① and one with a rectangular part ②. The flows are assumed to occur only in one direction z, and the flow velocities through the channel parts ① and ② are expressed as v_{z1} and v_{z2}, respectively.

In the case of a Newtonian fluid, its equation of motion and boundary conditions are as follows:

$$\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial z} \tag{4}$$

The boundary conditions are:

In the channel part ① $(0 \leq x \leq H)$

$$v_{z1}(x, 0) = V_z,\ v_{z1}(x, \sqrt{H^2 - x^2}) = 0,$$

In the channel part ② $\left(H - \frac{W}{2} \leq x \leq 0\right)$

$$v_{z2}(x, 0) = V_z,\ v_{z2}(x, H) = 0,\ \frac{\partial}{\partial x} v_{z2}\left(H - \frac{W}{2}, y\right) = 0$$

And, from the continuity of flow in the boundary

$$v_{z1}(0, y) = v_{z2}(0, y),\ \frac{\partial}{\partial x} v_{z1}(0, y) = \frac{\partial}{\partial x} v_{z2}(0, y),$$

$$\frac{\partial}{\partial y} v_{z1}(0, y) = \frac{\partial}{\partial y} v_{z2}(0, y) \tag{B.C.}$$

The principle of superposition is applied for solving Eq. (4) with (B.C.) and the total volumetric flow rate is expressed as the sum of the drag flow rate and pressure flow rate:

The equation of motion on the pure drag flow is

$$\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} = 0 \tag{5}$$

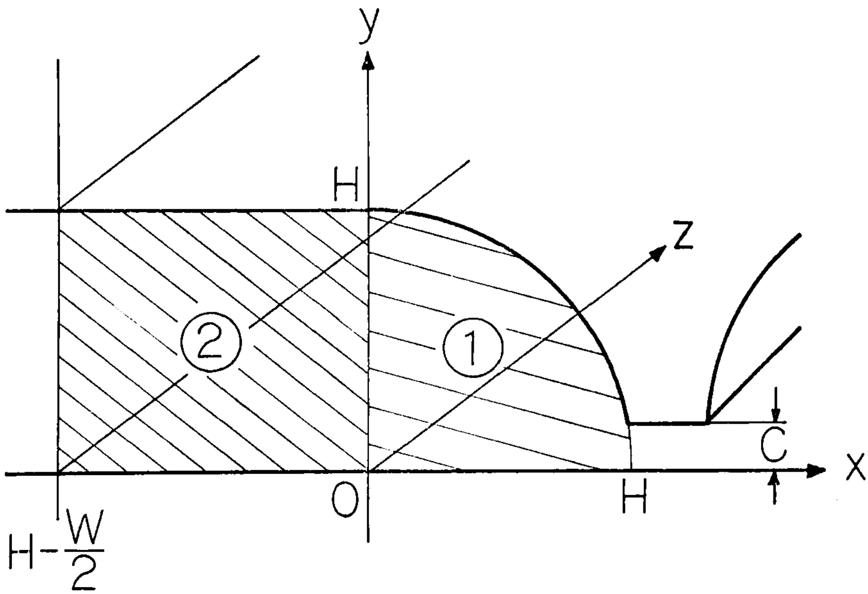

FIGURE 4 A rectangular channel having rounded corners; (1) a rounded corner channel, (2) a rectangular channel.

Solving this equation with (B.C.), the following equation is derived from the drag flow rate through the screw having m flights in parallel:

$$\alpha_v = 2m\left(\int_C^H \int_0^{\sqrt{H^2-y^2}} v_{z1}\,dx\,dy + \int_C^H \int_{H-W/2}^0 v_{z2}\,dx\,dy\right)\Big/N$$

where α_v is a geometric factor represented by W and H.

For the pure pressure flow, the equation of motion, Eq. (4), can be solved with the completely same boundary conditions as (B.C.) except $V_z = 0$. The geometric factor β_v can be derived from the pure pressure flow rate through the screw having m flights in parallel, provided that correction is made for the leakage flow in the flight clearance[16]:

$$\beta_V = -\frac{2m\mu}{\left(\frac{\partial P}{\partial z}\right)}\left(\int_0^H \int_0^{\sqrt{H^2-y^2}} v_{z1}\,dxdy + \int_0^H \int_{H-W/2}^0 v_{z2}\,dxdy\right) \times \left\{1 + \left(\frac{C}{H}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta\cdot\cos\theta}\right)^2\right\}.$$

In the same manner as with the rectangular flow channel [1], the volumetric flow rate extended for the modified non-Newtonian fluid is obtained:

$$\hat{Q} = \alpha_v N \pm \beta_v \dot{\gamma}^\circ \left(\frac{n}{n+1}\right)^{1-n} \left\{\frac{\sin\theta}{\eta^\circ \dot{\gamma}^\circ}\left(\frac{H}{2}\right)^{1-n}\right\}^{1/n} \left|\frac{\Delta P}{\Delta\lambda}\right|^{1/n} \tag{6}$$

where the sign in the second term on the right hand side is positive when $(\Delta P/\Delta\lambda) < 0$ and is negative when $(\Delta P/\Delta\lambda) > 0$

The power is obtained for the non-Newtonian fluid in the same manner as [1]:

$$\begin{aligned} \frac{\overline{\overline{HP}}}{t} = {} & \frac{m\eta^\circ \cdot \Delta\lambda}{\sin\theta}\left(\frac{\pi DN \cdot \cos\theta}{H\dot{\gamma}^\circ}\right)^{n-1} \\ & \times \left[(\pi DN)^2 \left\{\left(\frac{H}{C}\right)^{n-1}\left(\frac{e}{C}\right) - \frac{4W\sin^2\theta}{H}\right\} + \frac{\overline{\overline{HP_D}}}{t}\right] \\ & + \frac{m \cdot \Delta\lambda}{\sin\theta}\left(\frac{\overline{\overline{HP_P}}}{t}\right), \end{aligned} \tag{7}$$

where

$$\frac{\overline{\overline{HP_D}}}{t} = 2\pi\left\{\int_0^H \left.\frac{\partial V_{zD1}}{\partial y}\right|_{y=0} dx + \int_{H-W/2}^0 \left.\frac{\partial v_{zD2}}{\partial y}\right|_{y=0} dx\right\} DN \cdot \cos\theta,$$

$$\frac{\overline{\overline{HP_P}}}{t} = 2\eta^\circ\pi\left\{\int_0^H \left.\frac{\partial v_{zP1}}{\partial y}\right|_{y=0} dx + \int_{H-W/2}^0 \left.\frac{\partial v_{zP2}}{\partial y}\right|_{y=0} dx\right\} DN \cdot \cos\theta,$$

3 Power law flow between shallow parallel plates

Assuming the power law fluid, the following equation of motion on the flow between shallow parallel plates is obtained:

$$\frac{d}{d\overset{*}{y}}\left(\left|\frac{d\overset{*}{\Phi}_z}{d\overset{*}{y}}\right|^{n-1}\frac{d\overset{*}{\Phi}_z}{d\overset{*}{y}}\right) = \pm 1 \tag{8}$$

where

$$\overset{*}{y} \equiv y/H,\ \overset{*}{\Phi}_z \equiv v_z/\Gamma V_z,\ \text{and}\ \Gamma \equiv \frac{\dot{\gamma}^\circ H}{V_z}\left(\frac{H}{\eta^\circ\dot{\gamma}^\circ}\left|\frac{\partial P}{\partial z}\right|\right)^{1/n}$$

and the sign on the right hand side is positive when $(\partial P/\partial z) > 0$ and negative when $(\partial P/\partial z) < 0$. The boundary conditions are $\overset{*}{\Phi}_z(0) = 0$, and $\overset{*}{\Phi}_z(1) = 1/\Gamma$.

First, when $(\partial P/\partial z) = 0$, the volumetric flow rate can be easily obtained:

$$\hat{Q} = \alpha N, \tag{9}$$

where α is given by Eq. (2).

In the presence of $(\partial P/\partial z)$, the four velocity profiles are considered, as shown in Figures 5a–d.

In the case of Figure 5a, the volumetric flow rate can be expressed as[17]:

$$\hat{Q} = \pi WHDN\Gamma \cos\theta\left(\frac{n}{n+1}\right)$$
$$\left[\left(\frac{n}{2n+1}\right)\left\{(k_1+1)^{(2n+1)/n} - (k_1)^{(2n+1)/n}\right\} - (k_1)^{(n+1)/n}\right]$$

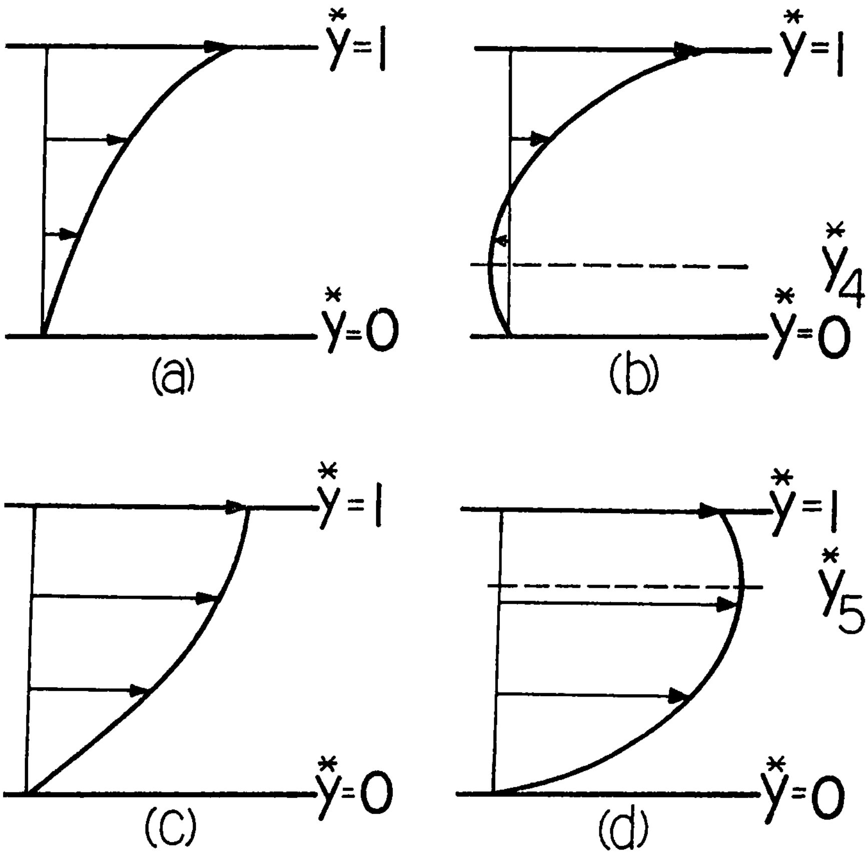

FIGURE 5 Types of velocity profiles in flow between parallel plates; when $(\partial P/\partial z) > 0$, a, b and when $(\partial P/\partial z) < 0$, c, d.

where k_1 is an integral constant given by:

$$(k_1)^{(n+1)/n} - (k_1+1)^{(n+1)/n} + \left(\frac{n+1}{n}\right)\frac{1}{\Gamma} = 0; \quad k_1 > 0$$

The correction on the flight clearance must be made in consideration of the leakage flow. In the solution it is not justified mathematically and physically to divide the volumetric flow rate into drag and pressure flow rates. However, assuming that the total flow rate minus the drag flow rate equals the pressure flow rate, it is possible to modify the drag and pressure flow rates with correction factors[16] and extend to the screw having m flights in parallel. Then the above equation is written as:

$$\hat{Q} = \alpha N + \beta_a \tag{10}$$

where

$$\beta_a = \pi^2 D^2 HN \sin\theta \cdot \cos\theta \left[\left(\frac{n}{n+1}\right)\Gamma\left[\left(\frac{n}{2n+1}\right)\{(k_1+1)^{(2n+1)/n} - (k_1)^{(2n+1)/n}\} - (k_1)^{(n+1)/n}\right] - \frac{1}{2}\right]$$

$$\times \left(1 - \frac{2C}{D} - \frac{em}{\pi D \sin\theta}\right)\left\{1 + \left(\frac{C}{H}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta \cdot \cos\theta}\right)^2\right\}F_P.$$

In the cases of Figures 5b–d,[18] the flow rates are written with the β_a in Eq. (10) replaced by β_b, β_c and β_d, respectively:

$$\beta_b = \pi^2 D^2 HN \sin\theta \cdot \cos\theta \left[\left(\frac{n}{n+1}\right)\Gamma\left[\left(\frac{n}{2n+1}\right)\left\{(\overset{*}{y}_4)^{(2n+1)/n} + (1-\overset{*}{y}_4)^{(2n+1)/n}\right\} - (\overset{*}{y}_4)^{(n+1)/n}\right] - \frac{1}{2}\right]$$

$$\times \left(1 - \frac{2C}{D} - \frac{em}{\pi D \sin\theta}\right)\left\{1 + \left(\frac{C}{H}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta \cdot \cos\theta}\right)^2\right\}F_p.$$

where

$$(\overset{*}{y}_4)^{(n+1)/n} - (1-\overset{*}{y}_4)^{(n+1)/n} + \left(\frac{n+1}{n}\right)\frac{1}{\Gamma} = 0; \quad 0 \le y_4 \le 1$$

$$\beta_c \equiv \pi^2 D^2 HN \sin\theta\cdot\cos\theta\left[\left(\frac{n}{n+1}\right)\Gamma\left[\left(\frac{n}{2n+1}\right)\{(k_6-1)^{(2n+1)/n} - (k_6)^{(2n+1)/n}\} + (k_6)^{(n+1)/n}\right] - \frac{1}{2}\right]$$

$$\times\left(1-\frac{2C}{D}-\frac{em}{\pi D\sin\theta}\right)\left\{1+\left(\frac{C}{H}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta\cdot\cos\theta}\right)^2\right\}F_p$$

where the integral constant k_6 is determined as

$$(k_6)^{(n+1)/n} - (k_6-1)^{(n+1)/n} - \left(\frac{n+1}{n}\right)\frac{1}{\Gamma} = 0;\quad k_6 > 1$$

And $\beta_d \equiv \pi^2 D^2 HN \sin\theta\cdot\cos\theta\left[\left(\frac{n}{n+1}\right)\Gamma\left[(\overset{*}{y}_5)^{(n+1)/n} - \left(\frac{n}{2n+1}\right)\{(\overset{*}{y}_5)^{(2n+1)/n} + (1-\overset{*}{y}_5)^{(2n+1)/n}\}\right] - \frac{1}{2}\right]$

$$\times\left(1-\frac{2C}{D}-\frac{em}{\pi D\sin\theta}\right)\left\{1+\left(\frac{C}{H}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta\cdot\cos\theta}\right)^2\right\}\cdot F_P$$

where

$$(\overset{*}{y}_5)^{(n+1)/n} - (1-\overset{*}{y}_5)^{(n+1)/n} - \left(\frac{n+1}{n}\right)\frac{1}{\Gamma} = 0;\quad 0 \le \overset{*}{y}_5 \le 1.$$

The power $\overline{\overline{HP}}$ is derived in the same way as [**1**] and [**2**].

In the case $(\partial P/\partial\lambda)$ is zero:

$$\frac{\overline{\overline{HP}}}{t} = \frac{m\eta^\circ(\dot\gamma^\circ)^2\cdot\Delta\lambda\cdot HW}{\sin\theta}\left\{\pi\left(\frac{D}{H}\right)\left(\frac{N}{\dot\gamma^\circ}\right)\right\}^{n+1}\left[(\cos\theta)^{n+1}\left\{1+\left(\frac{e}{C}\right)\left(\frac{H}{W}\right)\frac{1}{\cos^2\theta}\right\} + P_x\right],$$

$$P_x \equiv (\sin\theta)^{n+1}\left(\frac{n+1}{n}\right)^n(1-\overset{*}{y}_3)\left|\frac{1}{(1+\overset{*}{y}_5)^{(n+1)/n} - (\overset{*}{y}_3)^{(n+1)/n}}\right|^n,$$

$$\left(\frac{n}{2n+1}\right)\{(\overset{*}{y}_3)^{(2n+1)/n} + (1-\overset{*}{y}_3)^{(2n+1)/n}\} - (\overset{*}{y}_3)^{n+1/n} = 0;\quad 0 \le \overset{*}{y}_3 \le 1.$$

(*Equation 11 continued overleaf.*)

In the case of Figure 5a:

$$\frac{\overline{\overline{HP}}}{t} = \frac{m\eta^\circ(\dot{\gamma}^\circ)^2 \cdot \Delta\lambda \cdot HW}{\sin\theta}\left\{\pi\left(\frac{D}{H}\right)\left(\frac{N}{\dot{\gamma}^\circ}\right)\right\}^{n+1} \times \left[\Gamma^n(\cos\theta)^{n+1}(k_1+1)\left\{1+\left(\frac{e}{C}\right)\left(\frac{H}{W}\right)\frac{1}{\Gamma\cdot\cos^2\theta\cdot(k_1+1)^{1/n}}\right\}+P_x\right].$$

In the case of Figure 5b:

$$\frac{\overline{\overline{HP}}}{t} = \frac{m\eta^\circ(\dot{\gamma}^\circ)^2 \cdot \Delta\lambda \cdot HW}{\sin\theta}\left\{\pi\left(\frac{D}{H}\right)\left(\frac{N}{\dot{\gamma}^\circ}\right)\right\}^{n+1} \times \left[\Gamma^n(\cos\theta)^{n+1}(1-\overset{*}{y}_4)\left\{1+\left(\frac{e}{C}\right)\left(\frac{H}{W}\right)\frac{1}{\Gamma\cdot\cos^2\theta\cdot(1-\overset{*}{y}_4)^{1/n}}\right\}+P_x\right].$$

In the case of Figure 5c:

$$\frac{\overline{\overline{HP}}}{t} = \frac{m\eta^\circ(\dot{\gamma}^\circ)^2 \cdot \Delta\lambda \cdot HW}{\sin\theta}\left\{\pi\left(\frac{D}{H}\right)\left(\frac{N}{\dot{\gamma}^\circ}\right)\right\}^{n+1} \times \left[\Gamma^n(\cos\theta)^{n+1}(k_6-1)\left\{1+\left(\frac{e}{C}\right)\left(\frac{H}{W}\right)\frac{1}{\Gamma\cdot\cos^2\theta\cdot(k_6-1)^{1/n}}\right\}+P_x\right].$$

In the case of Figure 5d:

$$\frac{\overline{\overline{HP}}}{t} = \frac{m\eta^\circ(\dot{\gamma}^\circ)^2 \cdot \Delta\lambda \cdot HW}{\sin\theta}\left\{\pi\left(\frac{D}{H}\right)\left(\frac{N}{\dot{\gamma}^\circ}\right)\right\}^{n+1} \times \left[\Gamma^n(\cos\theta)^{n+1}(1-\overset{*}{y}_5)\left\{1+\left(\frac{e}{C}\right)\left(\frac{H}{W}\right)\frac{1}{\Gamma\cdot\cos^2\theta(1-\overset{*}{y}_5)^{1/n}}\right\}+P_x\right].$$

(11

4 Power law flow between shallow parallel plates including the effect of transverse flow

Considering a flow in a channel where $W \gg H$ and assuming that the two velocity components, v_x and v_z, are functions of y alone, the following equations with the effect of transverse flow are obtained[19]:

$$\left.\begin{aligned} \frac{d\overset{*}{v}_x}{d\overset{*}{y}} &= G_x(\overset{*}{y} - \overset{*}{y}_6)\{G_x^2(\overset{*}{y} - \overset{*}{y}_6)^2 + G_z^2(\overset{*}{y} - \overset{*}{y}_7)^2\}^{(1-n)/2n} \\ \frac{d\overset{*}{v}_z}{d\overset{*}{y}} &= G_z(\overset{*}{y} - \overset{*}{y}_7)\{G_x^2(\overset{*}{y} - y_6)^2 + G_z^2(\overset{*}{y} - \overset{*}{y}_7)^2\}^{(1-n)/2n} \end{aligned}\right\} \quad (12)$$

where $\overset{*}{y} = y/H,\ \overset{*}{y}_6 = y_6/H,\ y_7 = y_7/H,\ \overset{*}{v}_x = \dfrac{V_x}{\pi DN},\ \overset{*}{v}_z = \dfrac{v_z}{\pi DN}$

$$G_x = \phi^n\left(\frac{\Delta pH}{\Delta x}\right)\left(\frac{H\dot{\gamma}^\circ}{\pi DN}\right)^n,\quad G_z = \phi^n\left(\frac{\Delta pH}{\Delta z}\right)\left(\frac{H\dot{\gamma}^\circ}{\pi DN}\right)^n.$$

Assuming G_z, θ and n and estimating G_x, y_6 and y_7 in Eq. (12), it is possible to have approximate numerical solutions for the velocity components v_x and v_z by Runge-Kutta integrations.[19] The effects of flight, the leakage flow in the flight clearance and the screw having m flights in parallel are considered in the same manner as the previous chapter:

$$\hat{Q} = \alpha N + \beta_G \quad (13)$$

where
$$\beta_G \equiv \pi^2 D^2 HN \sin\theta\cdot\cos\theta\left(\frac{\overline{V}_z}{\pi DN\cdot\cos\theta} - \frac{1}{2}\right)$$
$$\times\left(1 - \frac{2C}{D} - \frac{em}{\pi D\sin\theta}\right)\left\{1 + \left(\frac{C}{H}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta\cdot\cos\theta}\right)^2\right\}F_P$$

and $\overline{V}_z$ is the average flow rate, which is

$$\overline{V}_z = \pi DN\int_0^1 \overset{*}{v}_z d\overset{*}{y}$$

The power is:

$$\frac{\overline{\overline{HP}}}{t} = \frac{\pi m\eta^\circ\dot{\gamma}^\circ DW\cdot\Delta\lambda\cdot N}{\sin\theta}\left\{\left(\frac{1}{\dot{\gamma}^\circ}\frac{dv_x}{dy}\right)^2\bigg|_{y=H} + \left(\frac{1}{\dot{\gamma}^\circ}\frac{dv_z}{dy}\right)^2\bigg|_{y=H}\right\}^{n-1/2}$$
$$\times\left\{\frac{\sin\theta}{\dot{\gamma}^\circ}\cdot\frac{dv_x}{dy}\bigg|_{y=H} + \frac{\cos\theta}{\dot{\gamma}^\circ}\frac{dv_z}{dy}\bigg|_{y=H} + \pi\left(\frac{De}{WC}\right)\left(\frac{N}{\dot{\gamma}^\circ}\right)\right\} \quad (14)$$

FLOW IN DIE SLIT

Polymer melt enters the die at $\lambda = 0$ and flows in the λ direction. Simultaneously, it flows through the die slit perpendicular to the λ axis. The flow in a slit can be regarded as a one dimensional flow between parallel plates because the pressure along the λ axis is nearly constant, the pressure gradient in the die screw is much smaller than that in the slit, and the depth of a slit sufficiently smaller than the width.

The total width of the die is Λ. The die is broken into M equal sections, each of width S, where

$$S = \Lambda / M.$$

Each section is further broken into J parts. Thus, the slit channel of continuously varying depth can be approximated.

The volumetric flow rate Q_i in the ith section of a die slit can be easily obtained

$$Q_i = \phi K_i \left(\frac{p_{i-1} + p_i}{2} \right)^{1/n} \tag{15}$$

where $p_i \equiv P_i / P_0$, P_i is the pressure at the exit of the ith section, ϕ is a dimensionless variable representing the flow characteristic of the melt, which is defined[5] as

$$\phi \equiv \left(\frac{P_0}{\eta^\circ \dot{\gamma}^\circ} \right)^{1/n} \tag{16}$$

Also, K_i represents the mobility of the melt in the die slit, and $1/K_i$ is a variable representing the resistance of the die slit and expressed as

$$\frac{1}{K} \equiv \left(\sum_{j=1}^{J} \frac{1}{A_j^n} \right)_i^{1/n} \tag{17}$$

where A_j may be given, in parallel plates having a depth I_j and length L_j, by

$$A_j \equiv \frac{S I_j^2 n \dot{\gamma}^\circ}{4n + 2} \left(\frac{I_j}{2L_j} \right)^{1/n} \tag{18}$$

and, for two sets of parallel plates, both sides tapered and either side tapered, having an entrance depth I_j, exit depth T_j, and length L_j, the following expressions apply respectively:

$$A_j \equiv \frac{S(I_j \cdot T_j)^2 n \dot{\gamma}^\circ}{(4n + 2)(I_j^{2n} \sim T_j^{2n})^{1/n}} \left(\frac{I_j \sim T_j}{L_j} \right)^{1/n}$$

$$A_j \equiv \frac{S(I_j \cdot T_j)^2 n \dot{\gamma}^\circ}{(4n + 2)(I_j^{2n} \sim T_j^{2n})^{1/n}} \left(\frac{(I_j \sim T_j)}{2L_j} \right)^{1/n}$$

DETERMINATION OF REDUCED PRESSURE

The volume balance of the polymer melt in the ith section is now considered. The melt flows into the die screw in the direction of the λ axis at a volumetric flow rate q_{i-1} and flows out of the section at a volumetric flow rate q_i, while, at the same time, flowing toward the die lip at a volumetric flow rate Q_i (Figure 2). The polymer melt flow in the direction of manifold axis mostly depends upon the drag force of the die screw in the screw die, while it only depends upon pressure difference in manifold die. Pressure in screw die is almost constant along the λ axis. Hence the flow in the die slit can be regarded as one in the die-lip direction perpendicular to the manifold. Thus, on the assumption that the interaction at the boundary between the screw and the slit can be neglected as well as various entry effects in the die slit,[5] the following equation is obtained:

$$Q_i = q_{i-1} - q_i \tag{19}$$

Further, from the volume balance of the polymer melt in the process of flow, we get

$$q_{i-1} = \sum_{i=i}^{M} Q_i. \tag{20}$$

Taking the volume balance with the volumetric flow rate along the screw in the ith section into account, and using the above equation, we obtain

$$\frac{1}{2}(q_{i-1} + q_i) = \frac{1}{2}\left(Q_i + 2\sum_{k=i+1}^{M} Q_k\right) = \hat{Q}_i \tag{21}$$

where $\hat{Q}_i$ is the average volumetric flow rate through the ith section of the screw.

Since P_0 is given and both the rheological properties of the molten polymer and the die dimensions are known, the unknown quantities are p_i's and Q_i's in the 1st to Mth, and q_i's in the 0th to (M-1)th. Thus, there are $3M$ unknown quantities. Because $3M$ independent equations can be generated from Eqs. (15), (20) and (21), this problem can be solved.

Introducing Eq. (15) into Eq. (21) yields

$$\frac{\phi}{2}K_i\left(\frac{p_{i-1} + p_i}{2}\right)^{1/n} + \phi\sum_{k=i+1}^{M} K_k\left(\frac{p_{k-1} + p_k}{2}\right)^{1/n} = \hat{Q}_i \tag{22}$$

Eq. (22) contains M unknown p_i's. However, a total of M simultaneous equations are obtained from Eq. (22) and M reduced pressures p_i's are determined with the consequence that all of the problems concerning the screw die can be completely solved.

UNIFORMITY

The overall volumetric flow rate Q_0 is equal to the sum of the volumetric flow rates Q_i's in all of the die sections:

$$Q_0 = \sum_{i=1}^{M} Q_i.$$

If n and η^0 are constant in those sections, Q_0 is introduced from Eq. (15):

$$Q_0 = \phi \sum_{i=1}^{M} K_i \left(\frac{p_{i-1} + p_i}{2} \right)^{1/n} \tag{23}$$

The relative deviation Δ_i from the average value may be expressed, from the Eqs. (15) and (23), as

$$\Delta_i = M \cdot \frac{K_i \left(\frac{p_{i-1} + p_i}{2} \right)^{1/n}}{\sum_{i=1}^{M} K_i \left(\frac{p_{i-1} + p_i}{2} \right)^{1/n}} - 1 \tag{24}$$

In order to indicate the uniformity in thickness of the sheet extruded using Δ_i values, a total of M uniformity values in the die sections must be arranged in parallel. For a comparison of the uniformity values, however, it is convenient to use a quantity which may be represented by a single numeral. For this purpose the standard deviation is obtained from the Δ_i values to define a uniformity function U[5] as

$$U = 1 - \sqrt{\frac{1}{M} \sum_{i=1}^{M} \Delta_i^2} \tag{25}$$

Eq. (24) indicates that Δ_i, and hence U, depends upon the geometrical configuration of the screw die and n. In practice, it is desired for the designing of an extrusion die, that U be kept at a high value with as little effect of n as possible. Then, the average uniformity $\bar{U}$ independent of n is defined as

$$\bar{U} = \int_0^1 U \, dn.$$

FOR COMPUTER ANALYSIS

The analysis of flow behavior of polymer melt in the screw die is accomplished by calculating the flow for each section by use of Eq. (22). Given the screw die dimensions and P_0, and knowing the flow characteristics of molten polymer, i.e., η°, $\dot\gamma^\circ$ and n, then A_i, K_i and ϕ can be calculated and only the reduced pressure p_i ($i = 1, 2, \ldots, M$) is left unknown. However, by the help of M equations imposed by Eq. (22) the values of p_i's can be determined.

The Flow equations and the power equations derived in [1] to [4] can be transformed into the forms used in broken section method, by the following replacement:

°$\hat{Q}$ is replaced by $\hat{Q}_i$.

°$\overline{\overline{HP}}$ is replaced by $\overline{\overline{HP}}_i$.

°Pressure gradient $|\Delta P/\Delta\lambda|$ is replaced by $P_0|p_{i-1} - p_i|/S$.

Since the total width Λ of die lip is broken into M equal sections, λ_i is written as

$$\lambda_i = S\left(i - \frac{1}{2}\right) = \Lambda\,(2i - 1)/2M.$$

In the case of [1] for example, Eq. (1) with $H_i = f(\lambda_i)$ is transformed to the following form

$$\hat{Q}_i = \alpha_i N \pm \beta_i\phi\dot\gamma^\circ\left(\frac{n}{n+1}\right)^{1-n}\left\{\left(\frac{H_i}{2}\right)^{1-n}\cdot\sin\theta\cdot\left|\frac{p_{i-1} - p_i}{S}\right|\right\}^{1/n} \tag{26}$$

where

$$\alpha_i = \frac{\pi^2 D^2 H_i\cdot\sin\theta\cdot\cos\theta}{2}\left(1 - \frac{2C}{D} - \frac{em}{\pi D\sin\theta}\right)\left(1 - \frac{C}{H_i}\right)^2\cdot F_{Di}$$

$$F_{Di} \equiv \frac{16W}{\pi^3 H_i}\sum_{g=1,3,5,\ldots}^{\infty}\left(\frac{1}{g}\right)^3\tanh\left(\frac{g\pi H_i}{2W}\right)$$

$$\beta_i \equiv \frac{\pi D H_i^3\sin\theta}{12}\left(1 - \frac{2C}{D} - \frac{em}{\pi D\sin\theta}\right)\left\{1 + \left(\frac{C}{H_i}\right)^3\left(\frac{W}{e}\right)\left(\frac{1}{\sin\theta\cdot\cos\theta}\right)^2\right\}\cdot F_{Pi}$$

$$F_{Pi} \equiv 1 - \frac{192 H_i}{\pi^5 W}\sum_{g=1,3,5\ldots}^{\infty}\left(\frac{1}{g}\right)^5\tanh\left(\frac{g\pi W}{2H_i}\right)$$

Introducing any of the transformed expressions for the case [1] to [4] into Eq. (22), we can obtain the p_i for each section, i.e., the pressure distribution along the λ axis. For example, introduction of Eq. (26) into the right hand side of Eq. (22) yields the following expression for p_i:

$$p_i = p_{i-1} \pm \frac{S}{\sin\theta}\left(\frac{2}{H_i}\right)^{1-n}\left[\left\{\frac{\alpha_i N}{\phi} - \frac{K_i}{2}\left(\frac{p_{i-1} + p_i}{2}\right)^{1/n} - \sum_{k=i+1}^{M} K_k\left(\frac{p_{k-1} + p_k}{2}\right)^{1/n}\right\}\frac{1}{\beta_i\dot\gamma^\circ}\left(\frac{n+1}{n}\right)^{1-n}\right]^n \tag{27}$$

where the sign of the second term in the right hand side is taken positive if $p_{i-1} - p_i > 0$ and $n = 1$, and negative if $p_{i-1} - p_i < 0$.

These equations are solved by the method of iteration and the p_i is obtained. In this method, assume an initial set $p_i^{(0)}$, and introduce these into the equations under consideration, Eq. (27) for example, and thereby obtain a second set $p_i^{(1)}$. Further introduce this $p_i^{(1)}$ into the equation just employed and obtain new pressure distribution $p_i^{(2)}$. Continue this process until the deviation between successive sets is less than some arbitrary specified amount ϵ, i.e., $|p_i^{(u)} - p_i^{(u-1)}| < \epsilon$. When this condition is realized, the iteration is regarded as convergent, and $p_i^{(u)}$ is considered to represent well the solution of the equation. The specified value of ϵ is recommended to choose about two orders smaller than the required accuracy. The method of iteration is particularly effective in solving the equations such as Eq. (27), but it may have a demerit deficiency in that the procedure sometimes fails to converge. Therefore, it is necessary to set an upper limit of iteration number, and stop the calculation if the solution does not converge after iteration up to this number. The success in the method of iteration depends mainly on the choice of the initial value.

Practical methods for improving the uniformity of extrudate are described in the following:

(*i*) *Ideal screw die* The ideal operating conditions and geometric parameters of the screw die are such that the pressure distribution along the λ axis is uniform, i.e., $p_i = p_0 = 1$, and the shape of the die slit is constant, i.e., $K_i = K_0 =$ constant. Eq. (22) can be reduced to a simpler form if the flow characteristics of polymer melt are known:

$$\hat{Q}_i = \Phi K_0\left(\frac{1}{2} + M - i\right) = \frac{Q_0}{M}\left(\frac{1}{2} + M - i\right)$$

The above equation gives,

$$\alpha_i = \Phi K_0\left(\frac{1}{2} + M - i\right)/N$$

and it is seen that the depth of the screw channel decreases almost linearly with the λ axis.

(*ii*) *Screw die shape of high uniformity* Given all of the die dimensions and the flow behavior of the polymer melt, p_i is obtained from Eq. (22), and Δ_i and U can be calculated. If the value of U obtained is not appropriate, the dimensions of the die slit or the die screw must be changed to improve the U so that U exceeds the desired value $\overset{*}{U}$.

(ii) 1: Method for improving the dimensions of die slit[5]

$\Delta_i = 0$ yields

$$K_i^{(u)} = \frac{\sum_{i=1}^{M} K_i \left(\frac{p_{i-1} + p_i}{2}\right)^{1/n}}{M\left(\frac{p_{i-1} + p_i}{2}\right)^{1/n}}$$

Calculate $K_i^{(1)}$ by introducing $K_i^{(0)}$ and $p_i^{(0)}$, which have been established from the dimensions of the die in section (i), into the above equation. Next, introduce $K_i^{(1)}$ into Eq. (22) to obtain new $p_i^{(1)}$. Repeating this process successively until a value of U larger than $\overset{*}{U}$ is obtained. In this case it is desirable that the values of $p_i^{(u)}$ from Eq. (22) are determined as independently on N as possible.

(ii) 2: Method for improving dimension of die screw

From $\Delta_i = 0$

$$\left(\frac{p_{i-1} + p_i}{2}\right)^{(u)} = \left[\frac{\sum_{i=1}^{M} K_i \left(\frac{p_{i-1} + p_i}{2}\right)^{1/n}}{MK_i}\right]^n$$

Calculate $p_i^{(1)}$ by introducing $p_i^{(0)}$ which have been calculated from the die dimensions given in section (i), into the above equation. In the same way described above, calculation is repeated until U exceeds $\overset{*}{U}$. Introducing the final values of $p_i^{(u)}$ into Eq. (22), we can determine the screw dimension as independently on N as possible.

DISCUSSION

A method with which the geometrical parameters of dies are to be improved and a method of computer analysis by means of iteration procedures was described paying attention to the uniformity of flow from a screw die.

The key point of the simulation in this analysis is dependent on the setting of initial values. Moreover, influence of frequency of screw rotation and flow index are also significant.

The most ideal shape of a screw die is that with $p_i = 1$, and with uniform shape of the die lip along the λ axis and the depth of screw channel decreases almost linearly apart from the die inlet.

Though the interactions between the screw flow and the slit flow was assumed to be neglected, in actual cases, there exists a pressure difference also along the direction parallel to the λ axis in the die slit. This phenomenon is particularly remarkable at the vicinity of the die inlet, hence it is desirable to analyze the flow two-dimensionally.[20]

The variable K_i is determined by both the shape of slit and the flow characteristics of the molten polymer but the K_i becomes dependent only on the shape of the slit if the flow characteristics are assumed to remain constant in all sections. The flow of polymer melt in the slit of which depth varies continuously along the flow direction of extrudate can be analyzed by taking an infinitesimal length of the parallel-plate channel. By increasing the number of plate channels, the model approaches an actual die and the accuracy of analysis is improved.

Pressure loss generated between the manifold and the die slit and between different depth slit channels can be neglected, since the velocity of polymer is slow and the depth difference is small.

A constant depth is preferable for die lips along the outlet, because of die swell. And the land length is to be adjusted so that the maximum shear stress at the wall is kept under the critical stress above which melt fracture occurs.

As is the case in extruder screws, both corners of the die screw channels are rounded for preventing degradation of polymer. The corner radius in the die screw is larger than that of extruders. The effect of the rounded corner may be neglected if the corner is sufficiently smaller than the channel depth, but if the depth is small enough as in the vicinity of the drive side, the effect of the corner on the flow seems to be quite large. Not only flow in the down channel direction but also a transverse flow should be considered, since polymer melt flows in a channel in a helical fashion. Further, molten polymer exhibits very complicated interactions between drag and pressure flows with non-Newtonian behavior and so it is not desirable that the flow equation derived for a Newtonian fluid is modified for a power law fluid.

While the flow equations derived in [**1**] to [**4**] are known to yield good approximation, hence, much better results are obtained by following analysis.

A power law fluid is assumed to flow in a rectangular channel with rounded corners on both sides, and the influence of transverse flow is considered. The coordinate axes are shown in Figure 4. The equations of motion are reduced to the form:

$$\frac{\partial P}{\partial x} = \frac{\partial \tau_{yx}}{\partial y}, \frac{\partial P}{\partial z} = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}$$

where

$$\tau_{yz} = \eta\left(\frac{\partial v_x}{\partial y}\right), \tau_{xz} = \eta\left(\frac{\partial v_z}{\partial x}\right) \text{ and } \tau_{yz} = \eta\left(\frac{\partial v_z}{\partial y}\right)$$

$$\eta = \eta^0(\dot{\gamma}^0)^{1-n}\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_z}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial x}\right)^2\right\}^{(n-1)/2}$$

The boundary conditions are:

in $0 \leq x \leq H$,

at $y = 0;\ v_{z1} = V_z,\ v_{x1} = -V_x$

at $y = \sqrt{H^2 - x^2};\ v_{x1} = v_{z1} = 0$

at $x = 0,\ v_{z1} = v_{x2},\ v_{z1} = v_{z2},\ \frac{\partial v_{x1}}{\partial x} = \frac{\partial v_{xz}}{\partial x},\ \frac{\partial v_{z1}}{\partial x} = \frac{\partial v_{z2}}{\partial x}$

$$\frac{\partial v_{x1}}{\partial y} = \frac{\partial v_{x2}}{\partial y},\ \frac{\partial v_{z1}}{\partial y} = \frac{\partial v_{z2}}{\partial y}$$

in $H - \frac{W}{2} \leq x \leq 0$,

at $y = 0;\ v_{z2} = V_z,\ v_{x2} = -V_x$

at $y = H;\ v_{x2} = v_{z2} = 0$

at $x = H - \frac{W}{2},\ \frac{\partial v_{z2}}{\partial x} = 0$

and

$$\int_0^H \int_0^{\sqrt{H^2-y^2}} v_{x1}\,dx\,dy + \int_0^H \int_{H-W/2}^0 v_{z2}\,dx\,dy = 0.$$

The velocity profile is obtained by numerical analysis of these equations.

Among the various flows the better results are when the equation above mathematically perfect can be solved, however, the equation derived in [1] is convenient as its easiness.

In the case of the die screw, most of the flow arises from the drag by the screw, and the pressure flow rather compensates the uniformity of the flow, and its contribution is relatively small compared to that of the drag flow. Therefore, since a curvature correction[21] for the pressure flow is negligibly small, we can adopt the simple correction factor only for a pure drag flow.[22]

The analysis has been carried out by assuming an isothermal flow, but an actual flow of molten polymers contains temperature gradients as a result of wall temperature and viscous dissipation, and such temperature gradient influences the flow strongly. Consequently the energy balance should be analyzed in considering with the temperature dependence of viscosity, viscous heating and wall temperature.

It has been assumed that the flow index n is constant at all points in the die, but n is a function of shear rate. Since the influence of n on flow is rather strong, it is desirable to take account of the variation with shear rate. A power law fluid, however, has characteristic that the flow pattern at low shear rate deviates from the true flow behavior. Therefore, n should be taken as half of that at the wall shear rate, since the high viscosity at low shear rate must be avoided.

If N is low, the conveying force also decreases and P_i drops. For compensating the conveying force, therefore, K_i has to be largely increased as the fluid flows apart from the die inlet. As N increases, the pressure distribution approaches its ideal value $p_i = 1$, and K_i is a constant value in each section. When N further increases, the conveying force becomes excessive, p_i increases and so K_i is to be small as the fluid flows apart from the die inlet.

The accuracy of analysis is improved as the number of sections, M, increases and the accuracy becomes worse as a result of calculation errors if M exceeds a critical value. In practice, $M = 50$ would yield fairly good results and $M = 100$ would be sufficient.

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References

1. J. F. Carley, *J. Appl. Phys.* **25**, 1118 (1954).
2. D. J. Weeks, *British Plastics*, May, 201 (1958).
3. J. R. A. Pearson, *Trans. Inst. Plastics Ind.* **30**, 230 (1962).
4. J. R. A. Pearson and F. E. Devine, *Rubber World*, Nov., 49 (1963).
5. J. M. McKelvey and K. Ito, *Polym. Eng. Sci.* **11**, 258 (1971).
6. H. Ozaki, *Japan Plastics Age*, Feb, 28 (1968).
7. H. Morohashi, T. Fujisawa and K. Endo, *Japan Plastics*, July, 21 (1970).
8. H. Morohashi, T. Sawada and F. Akutsu, *Japan Plastics Age*, Feb., 23 (1973).
9. H. Ozaki, Technical Information of IKEGAI Screw Die, Aug. (1972).
10. F. R. Eirich, *Rheology*, Vol. 3, p. 593, Academic Press (1960).
11. J. M. McKelvey, *Polymer Processing*, p. 250, John Wiley, New York, N. Y. (1962).
12. Ibid, p. 268.
13. Ibid, p. 252.
14. Ibid, p. 272.
15. Ibid, p. 273.
16. F. R. Eirich, *Rheology*, Vol. 3, p. 601, Academic Press (1960).
17. Z. Rotem and R. Shinner, *Chem. Eng. Sci.* **15**, 130 (1961).
18. F. W. Kroesser and S. Middleman, *Polym. Eng. Sci.* **5**, 231 (1965).
19. R. M. Griffith, *Ind. Eng. Chem. Fundamentals* **1**, 189 (1962).
20. M. Ishida and K. Ito, *Reports on Progress in Polym. Phys. in Japan*, XV, 101 (1972).
21. Z. Tadmor, *Polym. Eng. Sci.* **6**, 203 (1966).
22. J. M. McKelvey, *Polymer Processing*, p. 233, John Wiley, New York, N.Y. (1962).

Nomenclature

A = geometric parameter for the slit flow [cm³/sec]
C = flight clearance [cm]
D = inside diameter of manifold [cm]
e = width of die screw flight [cm]
F_D and F_p = shape factors for the drag flow and the pressure [-]
G = reduced pressure gradient [-]
H = distance from the root of die screw to the manifold surface [cm]
l = depth of die slit [cm]
J = number of parts [-]
K = geometric parameter for the slit flow [cm³/sec]
L = length of die slit [cm]
M = number of sections [-]
m = number of die screw channels in parallel [-]
N = frequency of die screw rotation [revolution/sec]
n = flow index (≤ 1) [-]
P = pressure [kg/cm²]
p = reduced pressure [-]
Q = volumetric flow rate [cm³/sec]
$\hat{Q}$ = volumetric flow rate in die screw channel [cm³/sec]
q = die screw volumetric flow rate from a section [cm³/sec]
S = width of a section [cm]
U = flow uniformity [-]
v = velocity [cm/sec]
W = width of die screw channel [cm]
x, y and z = Cartesian coordinates [cm]
α = shape parameter for the drag flow [cm³]
β = shape parameter for the pressure flow [cm⁴]
Γ = reduced pressure gradient [-]
$\dot{\gamma}^\circ$ = shear rate in the standard state [1/sec]
Δ = relative flow deviation [-]
η° = non-Newtonian apparent viscosity in the standard state [kg.sec/cm²]
θ = helix angle [radian]
Λ = total width of die in λ direction [cm]
λ = coordinate in the die screw axial direction [cm]
μ = Newtonian apparent viscosity [kg.sec/cm²]
ϕ = pressure parameter [-]
$\overline{HP}$ = power [PS]